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Commendatore, Pasquale; Currie, Martin

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The Cobweb, Borrowing and Financial Crises

Pasquale Commendatore^Ψ and Martin Currie*

Ψ University of Naples 'Federico II'

* School of Social Sciences, University of Manchester, UK

Abstract

Studies of non-linear cobweb models have failed to address a fundamental issue: whether the complex dynamical behavior displayed by such models is consistent with the survival of producers. This paper shows that where borrowing is unconstrained, as is implicitly assumed in standard cobweb models, borrowing results in financial crises. Incorporating constraints on borrowing is needed to salvage cobweb models. Industry performance (in terms both of profitability and of the incidence of bankruptcies) is highly sensitive to the nature of such credit restrictions.

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Corresponding author:

Pasquale Commendatore

University of Naples, Dipartimento di Teoria Economica e Applicazioni, Via
Rodinò 22, 80138 Napoli, Italy.

E-mail: commenda@unina.it

Tel: +39 081 253 7447

Fax: +39 081 253 7454

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The Cobweb, Borrowing and Financial Crises

1. Introduction

Since their introduction in the 1930s to explain fluctuations in agricultural production and prices in terms of sequential production readjustments, cobweb models have played a pivotal role in developments in economic dynamics. In the standard model, firms in a competitive industry produce a single homogeneous product; there is a well-defined production period, with the producers' activities being synchronized; producers base decisions on price expectations; and the market-clearing product price is established instantaneously at the end of each period. With the sole inter-temporal link being via price expectations, particular attention has been devoted to their formation. Indeed, it was in the context of cobweb models that adaptive expectations, rational expectations, expectations based on the mean of all past prices and heterogeneous expectations were first analyzed.

It is, however, curious and regrettable that in a context the very essence of which is that production takes time, very little attention has been paid to how producers finance their production activities and to the possibility of their becoming bankrupt. Typically it is assumed implicitly that producers can borrow or lend any amount at a given market rate of interest determined by the overall state of the economy. Certainly a 'perfect' financial capital market is a powerful simplification frequently invoked by economic theorists. In a cobweb model, it seemingly enables theorists to dispense with financial constraints on producer behavior and to concentrate on technological constraints. But it can be a very misleading simplification. Indeed, assuming that producers can borrow any amount at a given interest rate not only does not rule out bankruptcy but makes it particularly likely. Nor is bankruptcy ruled out by assuming that producers pay for inputs at the end of the production period. To

put the matter starkly, the possibility of bankruptcy is necessarily eliminated only if producers rely exclusively on their own financial capital to pay for inputs in advance.

Section 2 sets out the assumptions of our model. Section 3 examines the case where producers can borrow or lend freely at a given interest rate. Looking beyond the usual treatment of the dynamical behavior of price reveals a fundamental problem: borrowing results in bankruptcy. However, this paper is not simply intended as a challenge to standard non-linear cobweb models. In Section 4, following a brief consideration of the case where firms rely exclusively on their own financial capital, we explore the implications of banks limiting what they are prepared to lend to producers. We examine the cases where borrowing limits depend on the values of durable assets available for use as collateral and where they depend on producers' financial wealth levels.

2. Assumptions

There are N units of a homogeneous durable asset, denoted by L , that is specific to the industry and in perfectly inelastic supply (akin to Ricardian land). Since the ownership and use of 1 (and only 1) unit of L is required for participation in the industry,¹ there are, in any period, N producers, where N is sufficiently large so that each acts as if a price-taker for the product. Producers can acquire other inputs, but they must pay for these at the outset of the well-defined production period using their own financial capital, possibly supplemented by borrowed funds. At the beginning of period t (before entering into any commitments for the ensuing period), the representative firm's total wealth is

$$W_t = F_t + V_t \tag{1}$$

¹ This asset could, for example, be land or a farm. With appropriate (re-)interpretations of what follows, it could be a transferable license or permit required for participating in the industry.

where F_t is its net financial wealth and $V_t \geq 0$ is the market value of its unit of L . The firm's output for the t^{th} period is

$$q_t = q_f + q_{v,t} \quad (2)$$

where $q_f > 0$ is the output per period that would result from using solely its unit of L and where any extra output, $q_{v,t} \geq 0$, is achieved by the purchase of additional inputs. The cost function, which is invariant over time, is

$$c(q_{v,t}) = q_{v,t}^\alpha \quad (3)$$

where $\alpha > 1$, so marginal cost is increasing. The firm's *net* borrowing for period t is

$$B_t = q_{v,t}^\alpha - F_t \quad (4)$$

where $B_t < 0$ implies having bank deposits on which interest is received. The interest rate, r , on a loan for the duration of the production period is determined by the overall state of the economy and is invariant over time. The firms in the industry earn the same rate r on any bank deposits, so that r constitutes the marginal opportunity cost of the use of own funds in financing the production process.

Producers are motivated by the accumulation of wealth. At the beginning of period t , subject to any financial capital constraint, the representative firm maximizes its expected financial wealth at the beginning of period $(t+1)$ or, equivalently, maximizes its expected profit for period t . The firm's expected price for the output of period t , p_t^e , is based on adaptive expectations,

$$p_t^e = p_{t-1}^e + \gamma(p_{t-1} - p_{t-1}^e) \quad (5)$$

where $0 < \gamma \leq 1$ is the price expectations adjustment speed, with $\gamma = 1$ constituting naïve expectations. Expected profit for period t is

$$\pi_t^e = p_t^e q_t - (1+r) q_{v,t}^\alpha. \quad (6)$$

This definition of expected profit allows for the opportunity cost of own funds used to finance production but does not take account of the opportunity cost of the funds tied up in ownership of L . Once the producer is committed to participation in the industry in the current period, the cost of ownership of L constitutes a sunk cost, and π_t^e amounts to an expected quasi-rent accruing to the ownership of L .

Output is sold at the end of the period. For simplicity, we assume that the total expenditure, E , on the product of this industry is given and invariant over time, implying a unit elastic product demand curve. The market-clearing price, established instantaneously, is

$$p_t = \frac{E}{Nq_t} \quad (7)$$

so that $0 < p_t \leq \bar{p} \equiv E/(Nq_f)$. Since total revenue is invariant, the realized profit per firm is a strictly monotonically declining function of output:

$$\pi_t = p_t q_t - (1+r) q_{v,t}^\alpha = \bar{\pi} - (1+r) q_{v,t}^\alpha \quad (8)$$

where $\bar{\pi} \equiv E/N$ is the maximum profit achieved when each firm produces the minimum output q_f . The firm's income for period t is

$$y_t = rF_t + \pi_t. \quad (9)$$

Its financial wealth at the end of the period is

$$F_{t+1} = F_t + y_t = (1+r) F_t + \pi_t. \quad (10)$$

The simplest assumption that captures the notion that the market value of 1 unit of L depends on the long-term profitability of its ownership is that it is given by the present value of the receipt in perpetuity of the mean of the representative producer's past profits. That is,

$$V_{t+1} = \frac{1}{r(t+1)} \sum_{\tau=0}^t \pi_{\tau} \quad (11)$$

subject to $V_{t+1} \geq 0$. The representative firm's total wealth at the beginning of the next production cycle is then $W_{t+1} = F_{t+1} + V_{t+1}$.

3. Unconstrained Borrowing

Suppose initially that firms can borrow any amount at the going market interest rate. Maximizing expected profit requires that marginal cost equal the expected price:

$$(1+r)\alpha q_{v,t}^{\alpha-1} = p_t^e. \quad (12)$$

From (2) and (12),

$$q_t = q_f + q_{v,t} = q_f + \left(p_t^e\right)^{\frac{1}{\alpha-1}} \psi \quad (13)$$

where $\psi = \left[(1+r)\alpha\right]^{-\frac{1}{\alpha-1}}$. Using (5), (7) and (13) yields the map f :

$$p_t^e = f(p_{t-1}^e) = (1-\gamma)p_{t-1}^e + \frac{\gamma E}{N \left[q_f + \left(p_{t-1}^e\right)^{\frac{1}{\alpha-1}} \psi \right]}. \quad (14)$$

Given an initial expected price p_0^e , the future time path of expected price is uniquely determined by (14). The time paths of q_t , p_t and π_t are determined uniquely from that of p_t^e . With an appropriate initial condition, the time path of V_t can be determined from that of π_t . The decomposition that results from unconstrained borrowing means that the time paths of p_t^e , q_t , p_t , π_t and V_t do *not* depend on the initial financial wealth, F_0 . In contrast, the time paths for B_t , y_t , F_t , and W_t depend on F_0 .

A fixed point for the map f corresponds to a stationary state, where the representative producer is maximizing (expected) profit on the basis of a price expectation that is being

realized. The stationary values $(\bar{p}, \bar{q}_v, \bar{q})$ satisfy (i) $\bar{p} = \bar{p}^e$, (ii) $\bar{q} = q_f + \bar{q}_v = q_f + \bar{p}^{\frac{1}{\alpha-1}} \psi$, and (iii) $N \bar{q} \bar{p} = E$. Fig. 1 shows \bar{p} and the corresponding industry output, $N \bar{q}$. The stationary profit, $\bar{\pi} = \bar{\pi} - (1+r) \bar{q}_v^\alpha > 0$, constitutes a return to the ownership of L . In a thorough-going stationary state, $\bar{V} = \bar{\pi}/r$ and *pure* profit (taking account of the opportunity cost of the wealth tied up in the ownership of L) is zero. In a stationary state, everything is stationary except for financial wealth, bank deposits and income.

The fixed point is locally stable if $-1 < f'(\bar{p}) < 1$, where $f'(\bar{p})$ denotes the first derivative of f evaluated at the fixed point. Expressing it in terms of \bar{q}_v and \bar{q} confirms that $f'(\bar{p}) < 1$:

$$f'(\bar{p}) = 1 - \gamma - \frac{\gamma \bar{q}_v}{(\alpha-1) \bar{q}} < 1. \quad (15)$$

Therefore, the fixed point is stable if $f'(\bar{p}) > -1$, that is, if

$$\bar{q}_v / \bar{q} < (2 - \gamma)(\alpha - 1) / \gamma. \quad (16)$$

With naïve expectations, f is strictly monotonically decreasing: the higher is p_{t-1}^e , the higher is q_{t-1} and the lower is $p_t = p_t^e$. The system is attracted either to the fixed point (where $\bar{q}_v / \bar{q} < \alpha - 1$) or to a period-two cycle (where $\bar{q}_v / \bar{q} > \alpha - 1$). Fig. 2(a), based on $\alpha = 1.5$ and $\gamma = 1$, shows the map f corresponding to Fig. 1: the fixed point is repelling and the system is attracted to the depicted period-two cycle.² With adaptive price expectations, the possible long-term behaviors are considerably enriched. Fig. 2(b), based on $\alpha = 1.075$ and $\gamma = 0.4$,

² All diagrams and simulations assume $q_f = 1$, $r = 0.1$, $N = 1000$ and $E = 5000$. All simulations assume $p_0^e = 0.99 \bar{p}$.

illustrates a period-three cycle, the hallmark of complex dynamics. Fig. 3(a), based on $\alpha = 1.1$, shows the impact of the expectations adjustment speed, γ , on the long-term behavior of expected price. The fixed point is stable for sufficiently slow speeds. As γ increases through $\gamma^{bif} \cong 0.24$, where $f'(\bar{p}) = -1$, a sequence of period-doubling bifurcations occurs. For speeds between $\gamma^S \cong 0.381$ and $\gamma^E \cong 0.564$, intervals of chaos (a positive Lyapunov exponent in Fig. 3(b)) and of order (a negative Lyapunov exponent) are intermingled. Increasing γ above γ^E gives rise to a sequence of period-halving (period-doubling reversed), until a stable period-two cycle is generated at $\gamma \cong 0.712$. As γ increases towards 1, the amplitude of the period-two cycle increases.

It should be emphasized that our model involves normal assumptions about costs and demand and that, with the assumption of unconstrained borrowing, it constitutes a standard cobweb model. The map f belongs to the class of difference equations analyzed by Hommes (1994) involving adaptive expectations and non-linear but monotonic demand and supply.³ As $q_f \rightarrow 0$, the map f tends to a form similar to that analyzed by Onozaki et al. (2000), who assume naïve expectations but cautious adjustment to the output that maximizes expected profit. But what these studies ignore is whether the long-term behaviors implied by the models are consistent with the long-run financial viability of producers. When this issue is

³ For this class, the difference equations are differentiable and possess a first derivative less than 1. As a specific case, Hommes explores the properties of a map derived from an ‘S-shaped’ supply curve and a linear demand function. His map has similar properties to our map f (it has two critical points for some $0 < \gamma < 1$ and is strictly decreasing at $\gamma = 1$) and the qualitative properties of the dynamics are substantially identical. A crucial difference is that, since Hommes normalizes prices by using the inflection point of his supply curve as the new origin, his model does not permit an evaluation of profitability.

addressed, it becomes evident that there is a fundamental problem with standard cobweb models.

For our model, as shown in Fig. 3(c), long-run average profit declines monotonically as the speed γ increases beyond γ^{bif} . For $\gamma > \gamma^\pi \cong 0.257$, average losses are incurred, and they increase rapidly as γ increases towards the case of naïve expectations. Negative long-run average profits set off alarm bells suggesting non-viability. However, the question of long-run viability cannot be settled conclusively by examining average profits: *in general, negative average profits are neither necessary nor sufficient for financial crises to occur.*

To determine whether financial crises occur, we need to consider explicitly the behaviors of net borrowing and of financial wealth. We define γ^b as the expectations adjustment speed below which producers are always able to finance internally their desired input acquisitions and never have recourse to borrowing. We define γ^{fc} as the speed below which financial crises do not occur, where, provisionally, we define a financial crisis as arising when the representative firm's debt is increasing period after period. Since a financial crisis can only arise as a result of borrowing, $\gamma^{fc} \geq \gamma^b$. In fact, with *unconstrained* borrowing, financial crises occur sooner or later (i.e., $\gamma^{fc} = \gamma^b$). The latter critical speed depends *inter alia* on the representative producer's initial financial wealth. Assuming that the latter is just sufficient to cover the cost of producing the initial expected profit maximizing output (i.e., $F_0 = (q_0 - q_f)^\alpha$ where $q_0 = q_f + (p_0^e)^{1/(\alpha-1)} \psi$), simulations show that $\gamma^{fc} = \gamma^b \cong 0.277$ for the parameters on which Fig. 3 is based. Note well that $\gamma^{fc} = \gamma^b > \gamma^\pi$. That is, there is a range of speeds for which, even though average profit is negative, the representative firm does not borrow and cannot go bankrupt. In this case, the production losses are being subsidized by the receipt of positive net interest. Fig. 4 shows the impact on profitability and on the

occurrence of financial crises of varying both the speed γ and the cost parameter α . Region *I* corresponds to parameter combinations that result in stationarity with profit $\bar{\pi}$. In Region *II*, average profit is positive but below the stationary profit. In Region *III*, average profit is negative but firms do not borrow (and cannot go bankrupt). The boundary between Regions *III* and *IV* shows for each α the corresponding speed γ^b below which firms never borrow. In Region *IV*, firms do borrow and, sooner or later, they go bankrupt. From Fig. 4, for most parameter combinations for which the model exhibits complex behavior, firms engage in borrowing and, with no constraints on that borrowing, they go bankrupt.

The incidence of financial crises cannot simply be eliminated by a *ceteris paribus* increase in demand: increasing E (or reducing N) increases the intercept of the map f and is a destabilizing force. With cyclical or chaotic system behavior, average profit is less than the stationary profit, and fluctuations increase the likelihood of financial crises. Similarly, assuming a demand curve with a constant elasticity other than -1 does not alter our conclusions in any fundamental way.⁴

Before considering the nature and implications of borrowing constraints, we make two historical observations. First, whereas the notion of negative average pure profits over the long run may be disquieting to those brought up on the standard neoclassical theory of a

⁴ Extending the model to incorporate the distribution of part of the representative firm's income to shareholders would complicate further the relationships between average profits, borrowing and the occurrence of bankruptcies. The greater the proportion of income that is distributed, the lower the critical speeds at and above which borrowing and financial crises occur. With such distribution, there can be a range of speeds for which firms borrow regularly without going bankrupt (i.e., $\gamma^b < \gamma^{fc}$ for a given α). Furthermore, it is possible that the distribution of earnings may be sufficiently high that bankruptcies occur even though long-run average profit is positive (i.e., $\gamma^{fc} < \gamma^\pi$ for a given α).

perfectly competitive industry, it would not have been troublesome to Knight. In his classic work, *Risk, Uncertainty and Profit*, he advanced his strongly-held belief that “business as a whole suffers a loss” (1971, 365): he argued that entrepreneurs, motivated by the prospect of profits, actually realize negative pure profits on average, and they sustain this essentially through foregoing some of the opportunity costs on those financial or physical resources that they themselves supply to their businesses. Second, in certain respects, our challenge to standard non-linear cobweb models echoes a largely ignored challenge to the linear cobweb model by Buchanan in 1939. He argued that “neither perpetual fluctuation at a given amplitude nor expanding fluctuation is theoretically possible if the supply curve is a competitive supply curve as most writers apparently had in mind in their exposition of the doctrine” since “losses will inevitably exceed profits” (80-81).⁵ He notes: “On the special assumption that there is always a group of new producers willing to rush in and dissipate their capitals with each swing of the cycle, the theorem may perhaps be valid” (81).

4. Constrained Borrowing

Denoting the representative producer’s financial capital fund at the beginning of period t by K_t , where this comprises both own financial wealth and the maximum that the producer could borrow, the financial capital constraint on output is

$$q_{v,t}^\alpha \leq K_t. \quad (17)$$

Maximizing expected profit subject to (17) requires

$$q_t = q_f + q_{v,t} = q_f + \min \left\{ \left(p_t^e \right)^{1/(\alpha-1)} \psi; K_t^{1/\alpha} \right\} \quad (18)$$

⁵ In contrast to our model, Buchanan’s analysis encompasses two alternative interpretations of the industry supply curve, neither of which corresponds to the case of Ricardian increasing costs.

where $q_f + (p_t^e)^{1/(\alpha-1)} \psi$ is the output that would maximize expected profit in the absence of a financial constraint (see (13)) and $q_f + K_t^{1/\alpha}$ is the maximum output consistent with the financial constraint.

To accommodate the entry of a new cohort of producers to replace those that go bankrupt, it is necessary to specify precisely when firms are deemed bankrupt and what the financial position of entrants is. Our banks follow a simple rule: a firm is declared bankrupt if and only if it has a financial debt that is not diminishing. That is, the representative producer is deemed to be bankrupt at the beginning of period t iff

$$F_t \leq F_{t-1} < 0. \quad (19)$$

We assume that where durable assets are sold to a new producer cohort, the purchase exhausts the funds of the representative entrant so that there is no own financial capital left for acquiring additional inputs (i.e., $F_t = 0$ for a firm entering at the beginning of period t). The latter seems the least arbitrary assumption, and it implies at least that new firms get off to a good start since each produces q_f and receives the maximum profit in its first period.

The dynamical system is depicted in Fig. 5. The financial capital fund K_t , which necessarily depends on own financial wealth F_t , may or may not depend also on the value of the durable asset, V_t . Where K_t depends directly only on F_t , then p_t^e depends on p_{t-1}^e and on F_{t-1} ; and F_t depends on p_{t-1}^e and on F_{t-1} . Given an initial expected price p_0^e and an initial financial wealth F_0 , the future time paths of p_t^e , K_t , q_t , B_t , p_t , π_t , y_t and F_t are determined uniquely; with an appropriate initial condition, the time path of V_t can be determined from that of π_t .

The system's behavior differs from the case of unconstrained borrowing if and only if the financial capital constraint (17) impacts on the behavior of the representative producer. Since the constraint is never binding for parameter combinations in Regions I, II and III in Fig. 4, the system's dynamical behavior is necessarily the same as for the map f for unconstrained borrowing. Therefore, the interesting (γ, α) combinations are those in Region IV. For the latter Region, the decomposition that occurs with unconstrained borrowing breaks down. Since the constraint (17) shifts over time as financial wealth changes, the system's dynamical behavior is considerably more complicated than for unconstrained borrowing.

4.1 Pure Internal Finance

Suppose initially that producers must rely exclusively on their own financial capital. With pure internal finance,

$$K_t = F_t \geq 0. \quad (20)$$

This excludes any possibility of bankruptcy: a firm that cannot borrow never falls into debt. Fig. 6, based on $\alpha = 1.1$ to permit comparisons with Fig. 3, shows the behaviors of expected price and of long-run average profit. Comparing pure internal finance with unconstrained borrowing for $\gamma > \gamma^b$, the long-run behavior of expected price is not overtly very different. Over the chaotic region, the behavior of expected price appears rather more 'noisy' in Fig. 6, but the ranges of variation at any speed are similar. However, the crucial difference is not evident from the bifurcation diagrams. Whereas recurrent financial crises are inevitable with unconstrained borrowing for $\gamma > \gamma^b$, bankruptcies cannot occur with pure internal finance, notwithstanding the negative average profits. For example, for naïve expectations, the period-two cycle with unconstrained borrowing involves a firm lifetime of just two periods; in contrast, the period-two cycle with pure internal finance is consistent with the continuing

survival of firms. For *all* (γ, α) combinations in Region *IV* in Fig. 4, pure internal finance involves survival with negative average profits.

4.2 Credit Rationing

Typically, firms are able to borrow but their ability to do so is constrained. Banks, facing the risk that a borrower may fail to repay the interest and the principal, ration credit. Lending to producers in a wide variety of industries and facing asymmetric information, our banks follow behavioral rules that discriminate between prospective borrowers according to their balance sheets.⁶

A natural case to consider first is that where the producers' durable asset L provides collateral for loans. Specifically, suppose that a bank is prepared to lend a producer up to a limit of $V_t/(1+r)$. Provided that the value of L does not fall, the proceeds from its sale would cover both the principal and the interest, protecting the bank against default.⁷ However, in our model, this credit constraint results in the same dynamical behavior as for pure internal finance. The explanation is that, for those parameter combinations for which the firms' own financial capital is insufficient to finance desired input acquisition (i.e., for Region *IV* in Fig. 4), long-run average profits are negative; according to (11), the durable asset is effectively worthless (i.e., $V_t \equiv 0$) and cannot be used as collateral for a loan.

A more interesting possibility is that banks discriminate between producers according to their financial wealth levels. This would be equivalent to basing the limit on the own

⁶ In their macro-analysis of business cycles, Bernanke and Gertler (1989) examine the significance of the creditworthiness of borrowers being dependent on their net worth.

⁷ In their analysis of credit cycles, Kiyotaki and Moore (1997, 218) invoke a similar borrowing constraint. In their rational expectations model, agents have perfect foresight of future durable asset prices.

financial capital that producers risk in production, for example, where banks are prepared to ‘match’ the own funds invested by borrowers. Following Day (1967, 1994) and Day et al. (1974), suppose that banks are willing to lend up to a multiple θ of a producer’s own financial capital, where $\theta > 0$ reflects the degree of cautiousness of the banking community.⁸ The representative producer’s financial capital fund, including possible borrowed funds, is then

$$K_t = \begin{cases} (1+\theta)F_t & \text{for } F_t \geq 0 \\ 0 & \text{for } F_t < 0 \end{cases} \quad (21)$$

Note well that, since a firm in debt cannot borrow, the simple bankruptcy rule (19), plausible for an individual bank that lacks information about the industry, turns out to be a sensible one for the banking community as a whole. To see this, suppose that at the outset of period $(t-1)$, the representative firm was in financial debt. Unable to borrow, it produced q_f . With each firm supplying q_f to the market, each received the maximum profit $\bar{\pi}$. A failure to make any positive contribution to paying off its debt (i.e., $F_t \leq F_{t-1} < 0$) is equivalent to $\bar{\pi} \leq r|F_{t-1}|$. Since the receipt of the maximum profit $\bar{\pi}$ in the previous period made no contribution to paying off the debt, the firm’s financial position is irretrievable: if it continued in production, its debt would inexorably deteriorate period after period if $F_t < F_{t-1} < 0$ and would remain the same in the (fluke) case in which $F_t = F_{t-1} < 0$. Thus, by deeming firms to be bankrupt if they have made no contribution to paying off their debts, banks are rationally cutting their losses. In contrast, if firms did make some contribution over the previous period to paying off their debts (i.e., $\bar{\pi} > r|F_{t-1}|$), it would not pay banks to

⁸ Fixing limits to loans is a crucial component of banks’ portfolio management. See Cohen and Hammer (1972) and Walker (1997).

deem them to be bankrupt. Firms would continue to reduce the debts period by period until they are cleared.⁹

Compared with pure internal finance (which would correspond to $\theta = 0$), the ability to borrow results in much greater system volatility for $\gamma > \gamma^b$. Fig. 7, based on $\alpha = 1.1$, shows the behaviors of expected price and of long-run average profit for $\theta = 4$. A crude story would be as follows. A low output in period t results both in a high price and in a high profit. In turn, the high price results in a high expected price. Furthermore, the high profit enhances the producers' ability to borrow. The resulting high output in period $(t+1)$ gives rise to a loss. If this loss results in the producer being in debt, output in period $(t+2)$ is at its minimum level, with price and profit at their highest levels. This continues until the debt is cleared. That the ability to borrow can result in sustained periods of debt and of low outputs lies behind another striking feature of Fig. 7, namely, that, for $\theta = 4$, long-run average profits are positive. This particular increase in average profits (compared to pure internal finance and *a fortiori* to the case of unlimited borrowing) is acquired without much risk to banks, since, for $\theta = 4$ and for the assumed cost structure ($\alpha = 1.1$), financial crises occur only for a very few isolated speeds.

Fig. 8 shows the impact of γ and of α on borrowing, profitability and bankruptcy for selected values of the credit rationing parameter θ . For each (θ, γ, α) combination, average profits were calculated and the incidences of borrowing and bankruptcy were identified for

⁹ Since $\bar{\pi}/r$ is the maximum conceivable value for L , our bankruptcy condition implies that a bankrupt firm's total wealth cannot be positive. In other words, our bankruptcy condition is equivalent to postulating that the representative firm is deemed bankrupt if its (hypothetically) receiving the discounted present value of the future infinite stream of the maximum possible profits would not result in a positive total wealth.

$501 \leq t \leq 2000$. The interpretations of the colors are shown in Table 1.¹⁰ For example, dark green signifies that producers borrowed at least once, that long-run average profit was negative but that no bankruptcies occurred. Note first the relationship between Fig. 8 and Fig. 4. For (γ, α) combinations in Regions *I*, *II* and *III* in Fig. 4, producers rely solely on internal finance and cannot go bankrupt. Therefore, Regions *I*, *II* and *III* appear, respectively, as white, light blue and dark blue in Fig. 8 (the level of θ being irrelevant). The borrowing constraint only impacts (γ, α) combinations in Region *IV*. As seen above, for pure internal finance ($\theta = 0$), losses are incurred for Region *IV*, so the latter would be dark blue. In Fig. 8(a), where banks are prepared just to match a borrower's own funds ($\theta = 1$), firms take advantage of the opportunity to borrow, long-run average profits are negative but bankruptcies never occur. Increasing θ can increase average profitability but at a cost of a greater risk of financial crises. Thus, in Fig. 8(b) where $\theta = 4$, the beneficial impact of the higher θ is manifested in the light green areas, indicating positive average profits; the hazards are reflected in the incidences of bankruptcies signified by the purple and red areas. Further increases in θ increase the likelihood of financial crises. Where bankruptcies are avoided, higher average profits are accompanied by increased variability of profits and possibly by the representative producer being frequently in debt. For $\theta = 10$ in Fig. 8(c), the red areas confirm the increased frequency of bankruptcies, while the yellow areas signify that, for some (γ, α) combinations, firms not only survive but also earn long-run average profits above the stationary profit.¹¹ Barely perceptible incidences of orange mean that it is

¹⁰ The colors are visible in the online version.

¹¹ This is consistent with Huang (1995), who shows that, under certain circumstances, 'cautious' responses by firms to fluctuating prices may result in long-run average profit above the stationary profit. Such responses

possible for bankruptcies to occur even though average profits exceed the stationary profit. For yet more lax credit limits, borrowing almost invariably results in bankruptcy. As Section 3 confirmed, for unconstrained borrowing, all (γ, α) combinations in Region 4 would be red, signifying borrowing leading to losses and to bankruptcy.

5. Some Concluding Comments

In reality, producers are constrained in their ability to borrow. In reality, producers go bankrupt. Our borrowing constraints and our bankruptcy condition presuppose that the banking community follows very simple behavioral rules. Our model could be extended by allowing credit limits to depend on the history of repayment defaults in this industry; by assuming that the rate of interest depends on the amount borrowed; or by introducing heterogeneity in the financial wealth levels of producers. Such amendments would surely reinforce our central conclusion: industry performance (in terms both of profitability and of the incidence of bankruptcies) is highly sensitive to the nature and degree of credit restrictions.

However simple the behavioral rules of our banks, they are certainly more plausible than the assumption (implicit in standard cobweb models) that banks are prepared to lend any amount to a producer, even to one that is falling further and further into debt. An implication of our model, which involves standard assumptions about costs and demand, is that unconstrained borrowing results in bankruptcies. To put our challenge to the standard non-linear cobweb model bluntly, a model designed to explain how prices and quantities can

involve upper bounds on the growth rates of output, which Huang suggests might be attributable to “capacity constraints, financial constraints and cautious response to price uncertainty by firms” (261).

fluctuate endogenously is methodologically unsatisfactory if it is inconsistent with the survival of producers for precisely those parameters that result in complex dynamical behavior.

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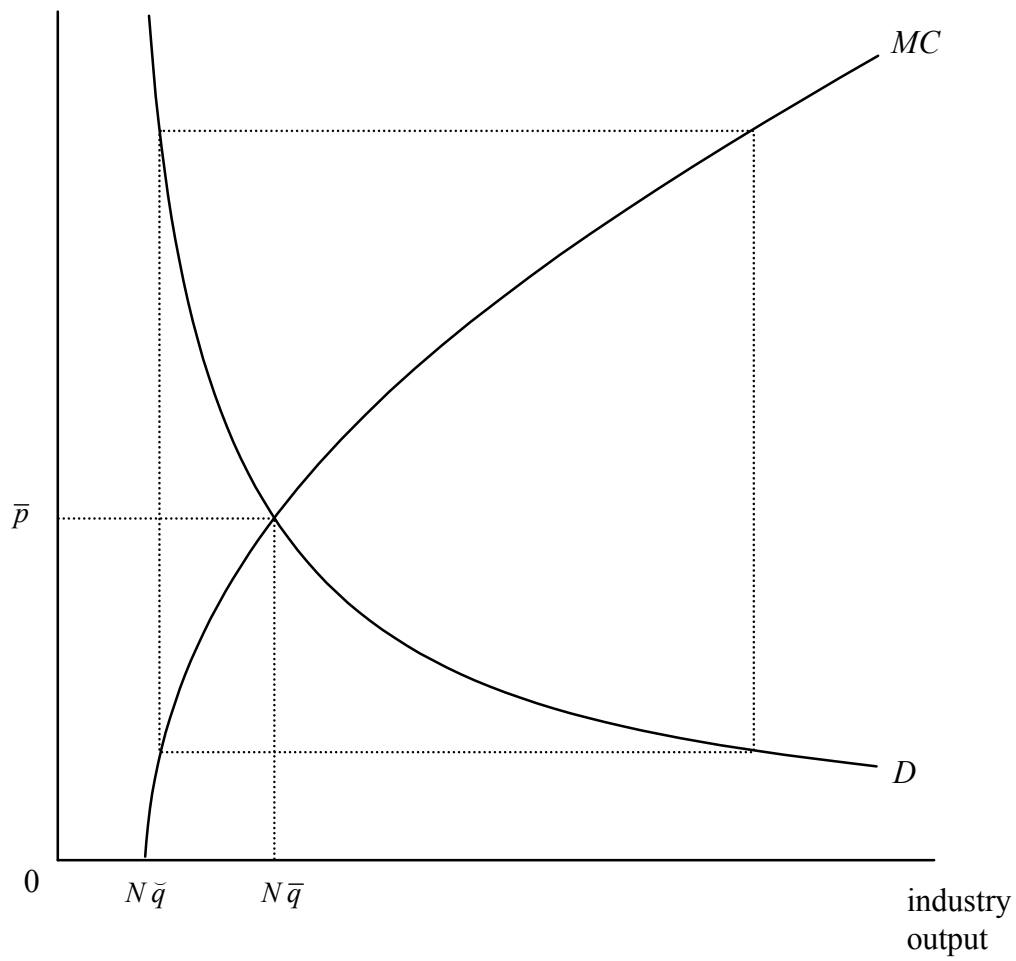
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	Borrowing	Average Profit ($av.$)	Bankruptcy
white	✗	$av. = \bar{\pi}$	✗
light blue	✗	$0 \leq av. < \bar{\pi}$	✗
dark blue	✗	$av. < 0$	✗
yellow	✓	$av. \geq \bar{\pi}$	✗
light green	✓	$0 \leq av. < \bar{\pi}$	✗
dark green	✓	$av. < 0$	✗
orange	✓	$av. \geq \bar{\pi}$	✓
purple	✓	$0 \leq av. < \bar{\pi}$	✓
red	✓	$av. < 0$	✓

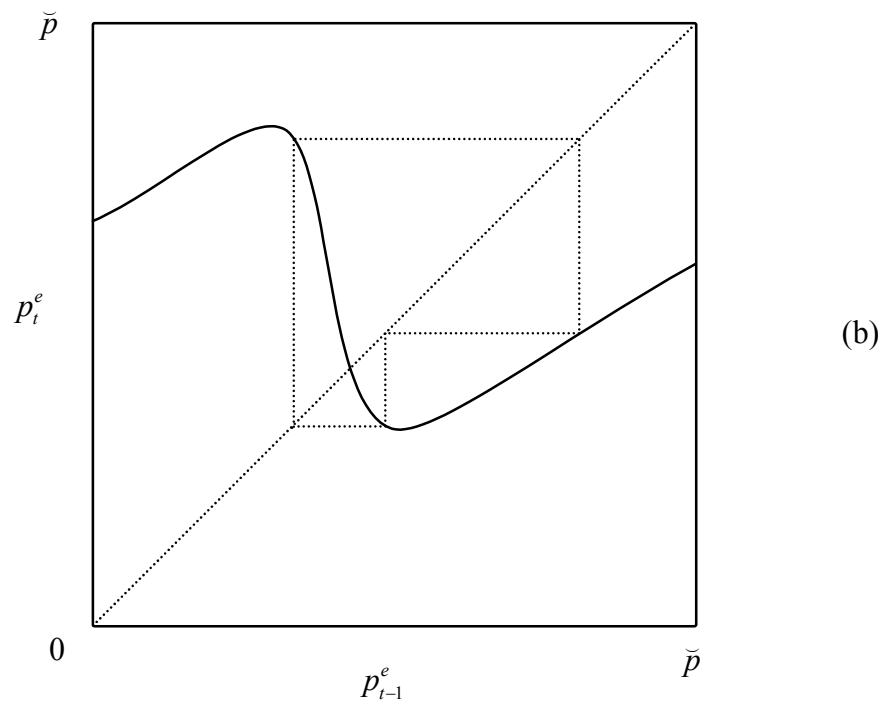
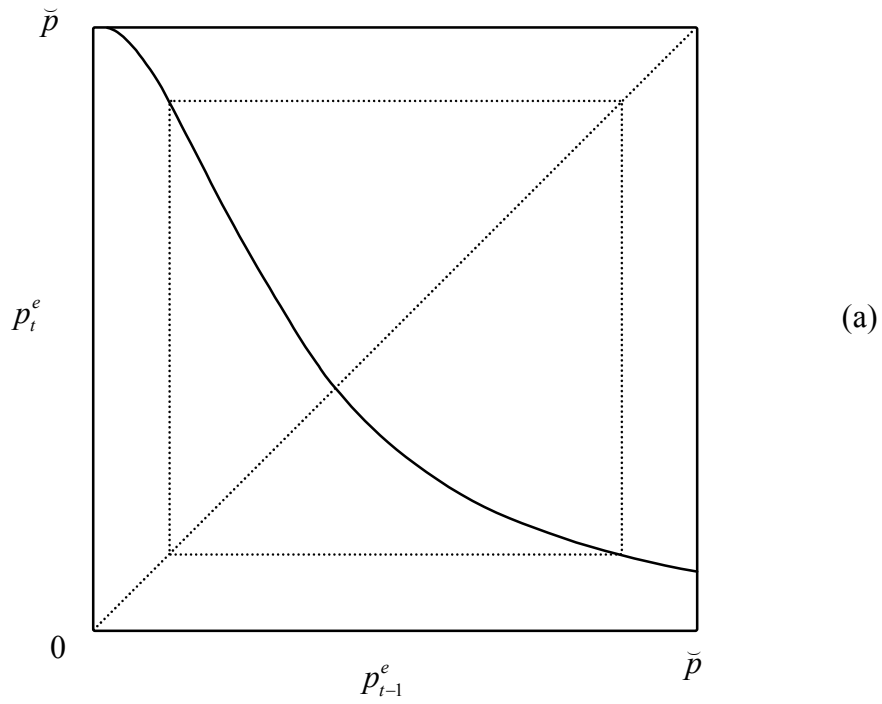
Table 1

Figure 1



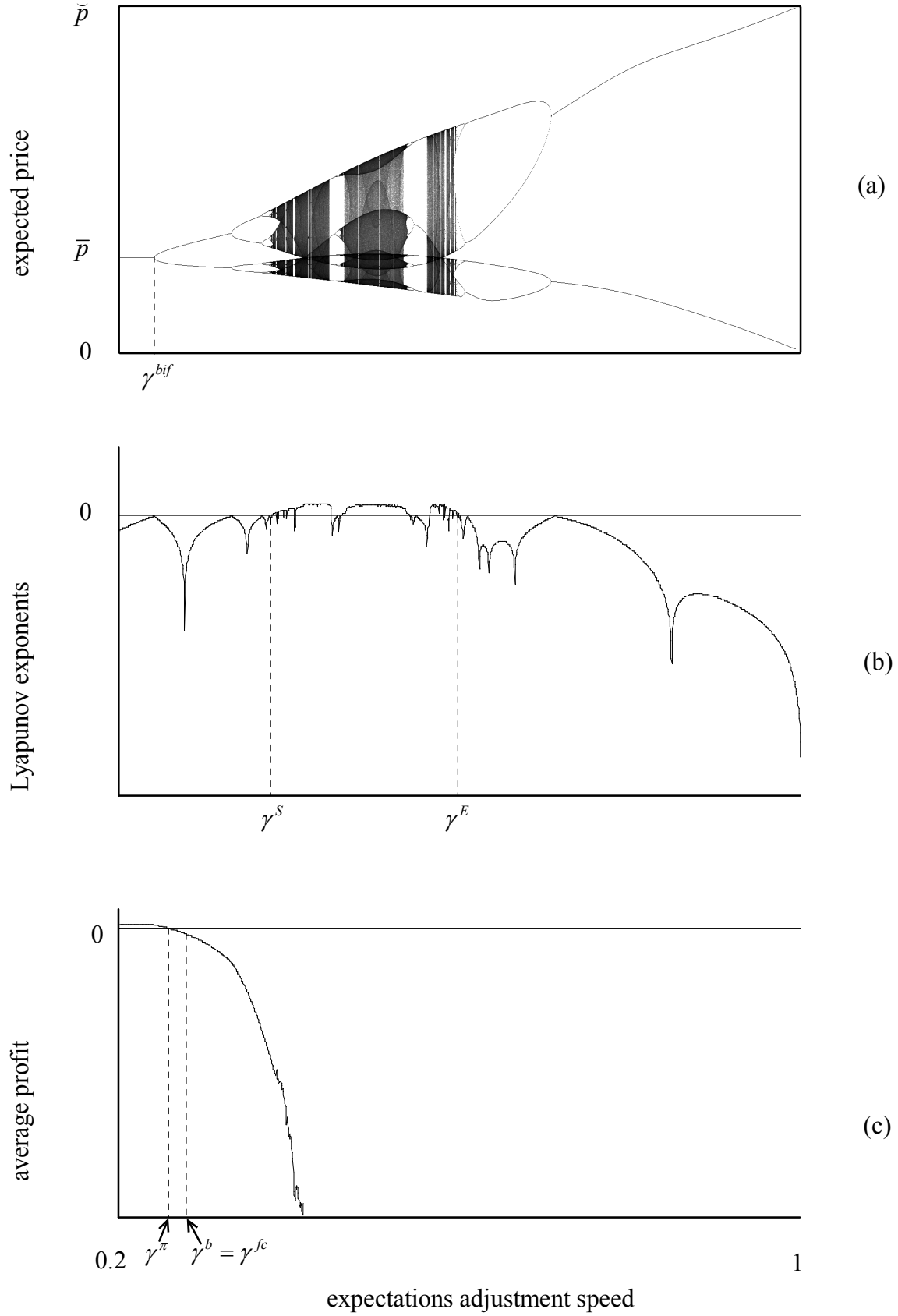
Market demand and industry marginal cost, based on cost parameter $\alpha = 1.5$. The stationary market price and industry output are \bar{p} and $N\bar{q}$. Naïve expectations give rise to a period-two cycle.

Figure 2



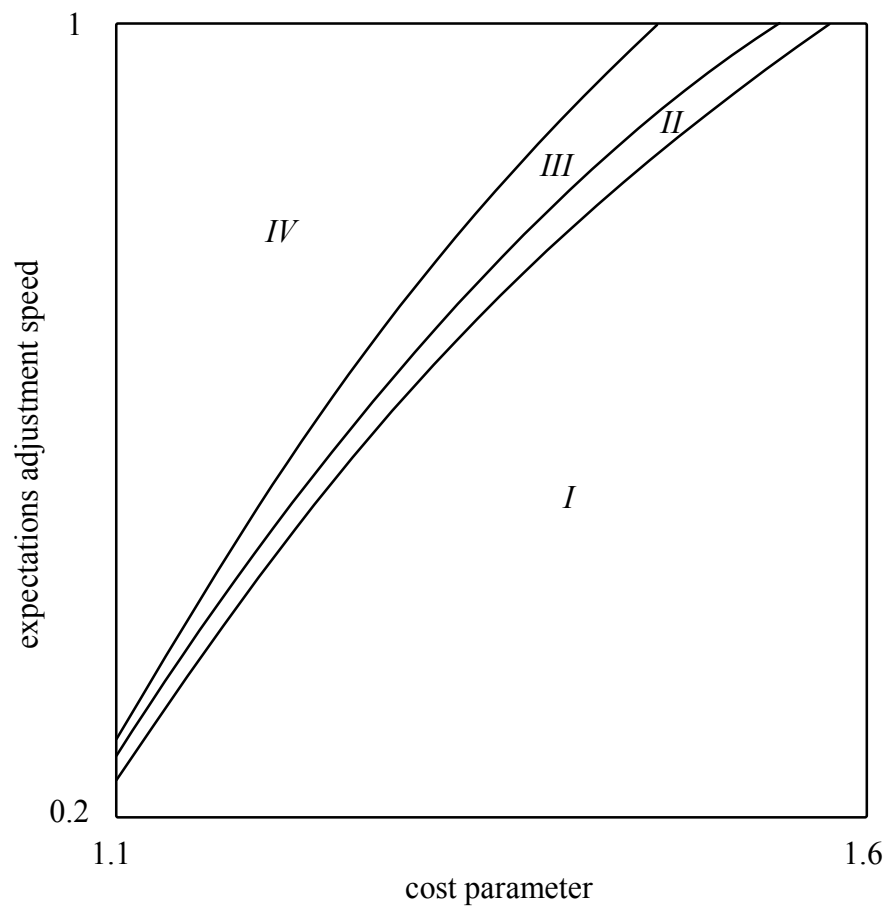
- (a) Attracting period-two cycle for the map f for $\alpha = 1.5$ and $\gamma = 1$.
 (b) Attracting period-three cycle for $\alpha = 1.075$ and $\gamma = 0.4$.

Figure 3



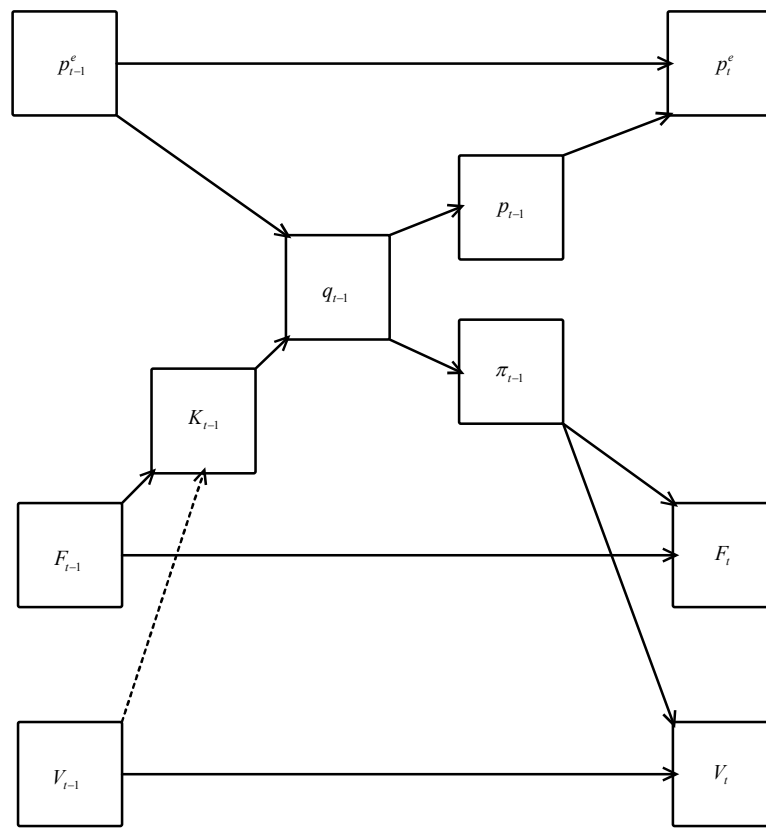
(a) Bifurcation diagram, based on $\alpha = 1.1$, that shows the dependence of the long-term behavior of expected price on the expectations adjustment speed, for $0.2 \leq \gamma \leq 1$, with unconstrained borrowing. (b) The corresponding Lyapunov exponents. (c) The corresponding long-run average profit.

Figure 4



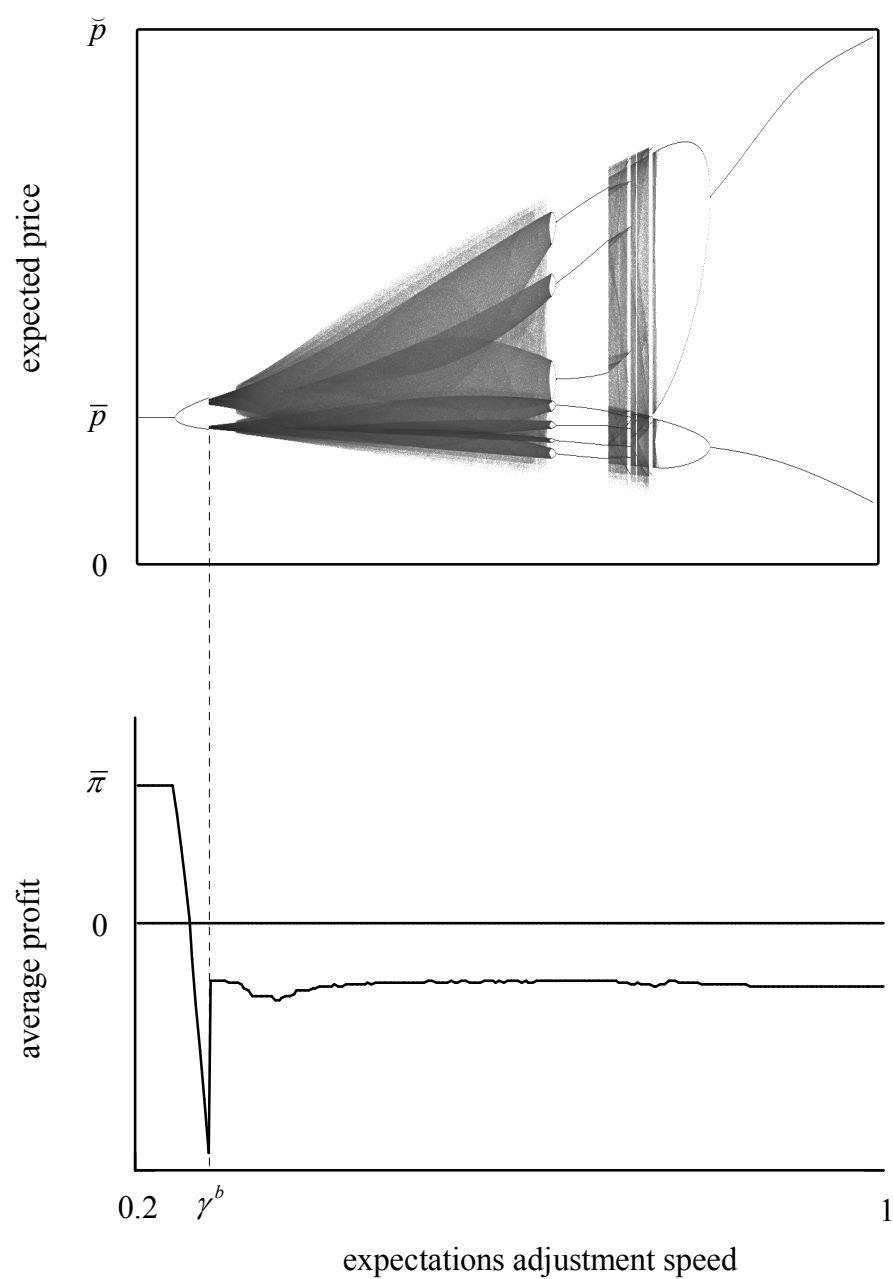
The impact on profitability and on the occurrence of financial crises of the expectations adjustment speed and the cost parameter with unconstrained borrowing. Region *I* involves stationarity. In region *II*, firms do not borrow and average profit is positive but below the stationary profit. In region *III*, firms do not borrow and average profit is negative. In region *IV*, firms borrow and go bankrupt.

Figure 5



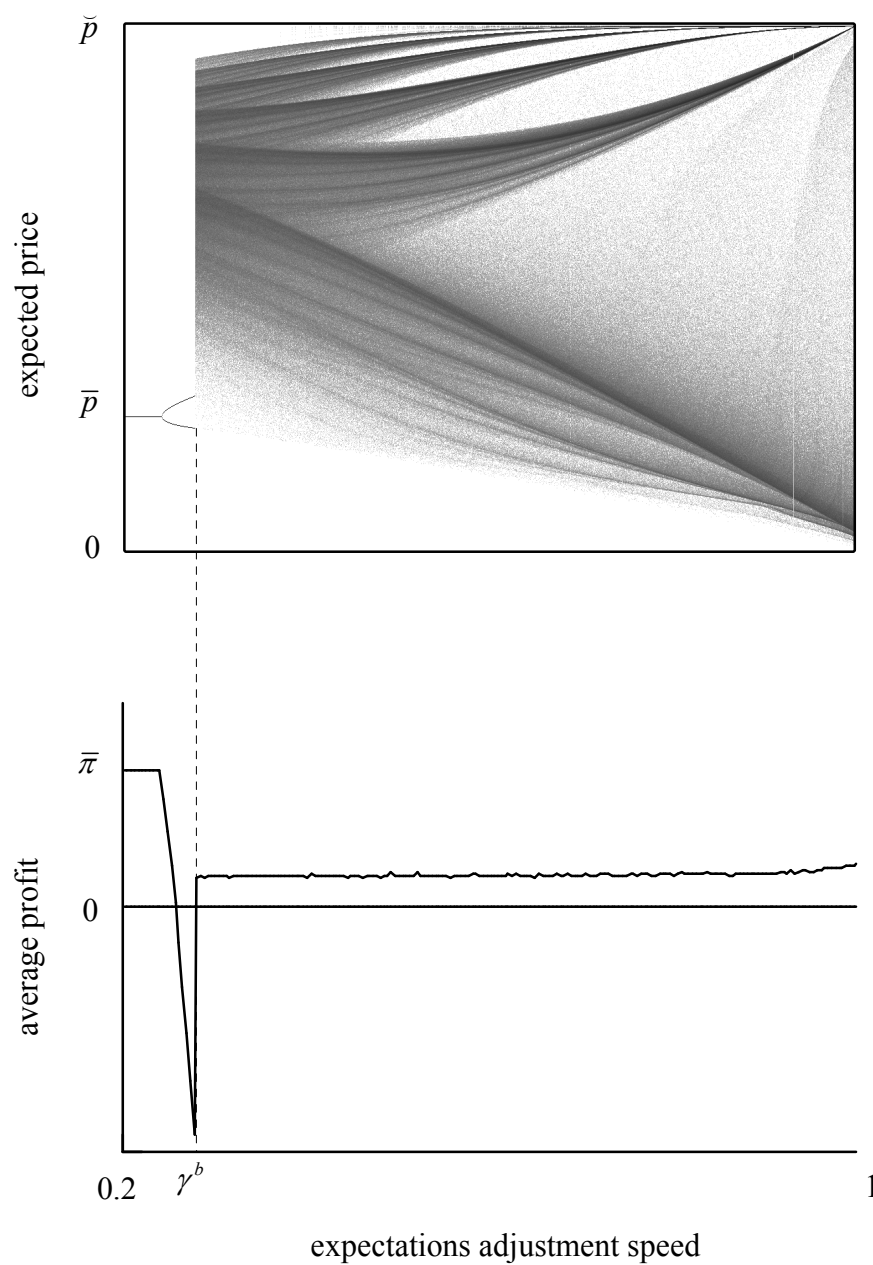
Dynamical system with constrained borrowing

Figure 6



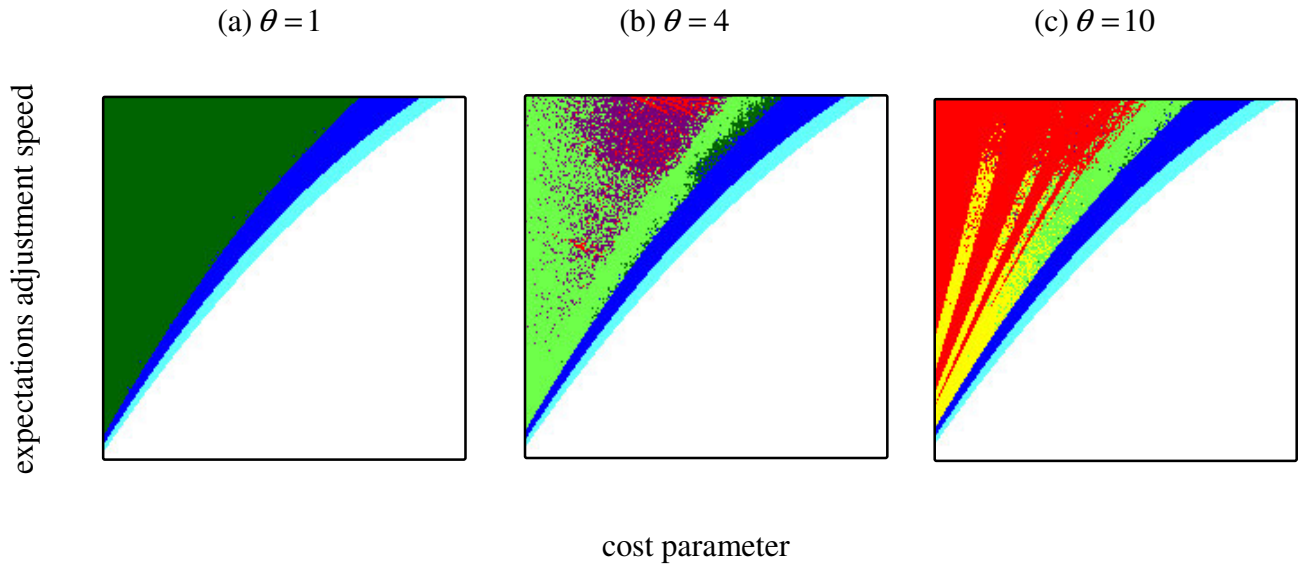
Bifurcation diagram and average profit, based on $\alpha = 1.1$, for pure internal finance.

Figure 7



Bifurcation diagram and average profit, based on $\alpha = 1.1$, for constrained borrowing with credit rationing parameter $\theta = 4$.

Figure 8



The impact of the expectations adjustment speed, $0.2 \leq \gamma \leq 1$, and of the cost parameter, $1.1 \leq \alpha \leq 1.6$, on long-run profitability, borrowing and bankruptcy for different values of the credit rationing parameter θ . White signifies stationarity; light blue signifies no borrowing and a positive average profit below the stationary profit; dark blue signifies no borrowing and a negative average profit; yellow signifies borrowing (but no bankruptcies) with an average profit above the stationary profit; light green signifies borrowing (but no bankruptcies) with a positive average profit below the stationary profit; dark green signifies borrowing (but no bankruptcies) with a negative average profit; orange signifies an average profit above the stationary profit but with bankruptcies; purple signifies a positive average profit below the stationary profit but with bankruptcies; red signifies a negative average profit with bankruptcies.